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## Beware of "Gaps" in Students' Fraction Conceptions

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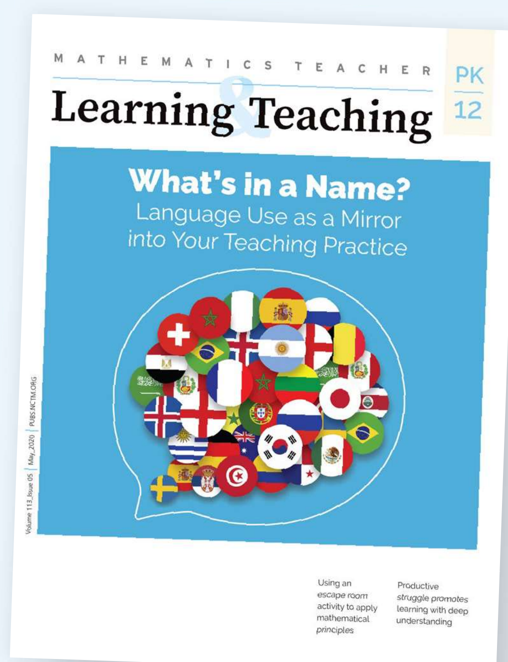


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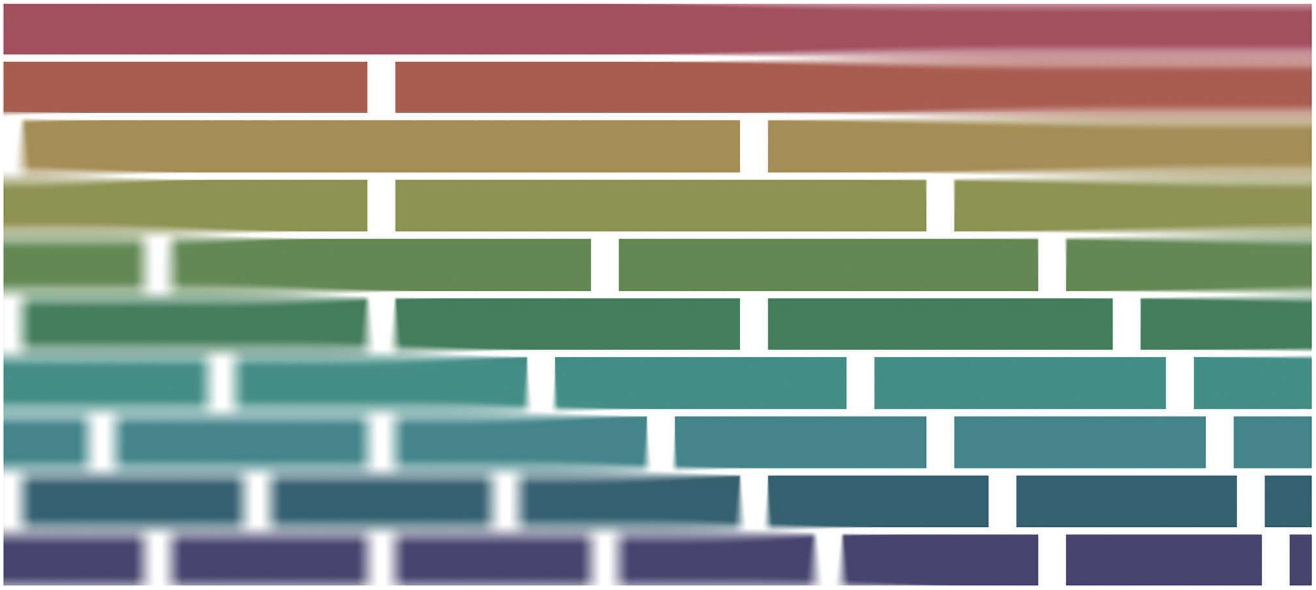


# Beware of “Gaps” in Students’ Fraction Conceptions

Many students have a dominant part-whole conception of fractions. We examine why this is problematic and explore strategies to move students beyond this limitation.

Patrick L. Sullivan, Joann E. Barnett, and Kurt Killion

**Before reading any further,** we invite you to engage your students in the following task: “Two pieces of licorice are the same size. Carlos ate 5/6 of a whole piece of licorice, and Terrell ate 7/8 of a whole piece of licorice. Who ate more licorice or did they eat the same?” Encourage your students to compare the two fractions by drawing a picture or explaining how they are thinking about the problem without using a common denominator procedure.

If several of your students respond, “Carlos and Terrell ate the same amount,” do not be alarmed. We have asked this question or a variation of the question to more than 500 students in Grades 4–7, as well as to university freshmen students as part of a diagnostic assessment. Regardless of the setting, the percentage of students who indicated that “Carlos and Terrell ate the same amount” is typically 25% or higher.

The focus of this article is three-fold: (1) to explain the fraction conception that underpins this type of reasoning, (2) to explain why this fraction conception can be problematic, and (3) to provide two instructional recommendations that support teachers in moving students beyond this reasoning and/or limit the potential of it happening.

## WHY ARE STUDENTS REASONING THIS WAY?

The use of the fraction pairs 5/6 and 7/8 as an initial probe into students' conception of fractions is intentional for two reasons. First, it is a task that sheds light on the nature of students' current fraction conceptions, especially those who reason that the fraction pairs are equal. Second, the problem provides a target to advance students' fraction conceptions toward because it challenges them to reason using the size of the unit.

It is important that we distinguish between two types of reasoning that we often see students use, *gap* and *missing piece*, and one that we aspire for our students to reach, *residual*. Each of these types of reasoning are underpinned by a different conception of fractions. Students who use *gap reasoning* to compare fractions are guided by a whole number conception of fractions, while those using *missing piece* utilize a part-whole conception of fractions. Meanwhile, those students who engage in *residual reasoning* to compare fractions utilize a fraction-as-measure conception.

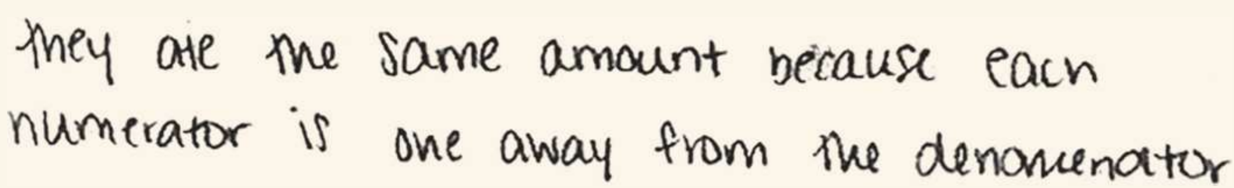
### Gap Reasoning

As shown in Figure 1, some students "see" the fractions 5/6 and 7/8 as a relationship between whole numbers. More specifically, they see the "gap" between each pair of whole numbers (e.g, 5 + **1** = 6 and 7 + **1** = 8) and use that relationship (**1** = **1**) to compare the two mathematical entities that we would call fractions. This reasoning has been described as *gap reasoning* (Clarke & Roche, 2009; Sullivan & Barnett, 2019).

### Missing Piece

Other students see fractions in terms of a set of actions, but they fail to attend to the size of the unit of the fractions in their reasoning. The work shown in Figure 2 exemplifies a student who associated fraction notation with the actions of partitioning the whole into a quantity of equal pieces represented by the denominator and shading the quantity of pieces represented by the numerator. Students who see fractions in this way are often able to correctly model fractions, but their reasoning to compare fractions only attends to the quantity of "missing pieces" to complete the whole (see Figure 2). Their conception of fraction involves seeing quantities because the denominator represents a quantity of pieces to partition the whole and not a size

***Figure 1*** A Student Uses Gap Reasoning to Compare 5/6 and 7/8

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of unit (i.e., *sixths* and *eighths*, respectively). This reasoning exemplifies a *part-whole* conception of fractions (Behr et al., 1983; Wilkins & Norton, 2018).

## MOVING STUDENTS FORWARD FROM THESE CONCEPTIONS

### Residual Reasoning

Our instructional goal is to move students beyond these conceptions of fractions toward one in which they would be able to reason about the size of the unit to compare fractions such as 5/6 and 7/8, as well as many others. This requires students to have two understandings. First, they need to be able to identify the size of the unit represented by each fraction (i.e., *sixths* and *eighths*). Second, they need to be able to engage in *residual reasoning* (Sullivan & Barnett, 2019). That is, they need to see the "missing piece(s)" in terms of a size of unit. For example, comparing the fractions 5/6 and 7/8, a student would reason that while 5/6 and 7/8 are both a quantity of 1 from the whole, there is a different size of unit (i.e., sixths and eighths) attached to that quantity (i.e., 1/6 and 1/8, respectively).

Figures 3 and 4 show student work exemplifying the use of residual reasoning to compare 5/6 and 7/8—using a slightly different different context than the one at the beginning of the article. Conceptual fraction comparision strategies—such as residual reasoning—are important because these strategies have been been associated with students' overall mathematical achievement (Siegler & Pyke, 2013; Siegler et al., 2011).

***Figure 3*** Student A Explanation Exemplifing Residual Reasoning

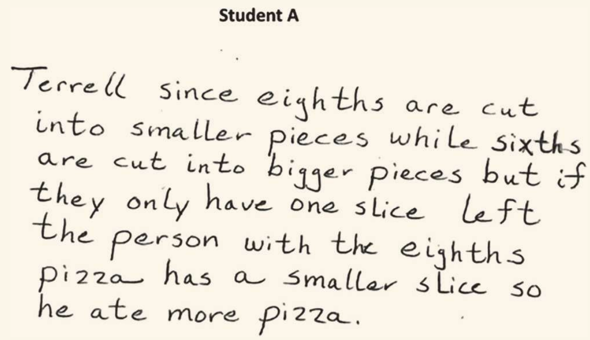



***Figure 4*** Student B Explanation that Exemplifies Residual Reasoning

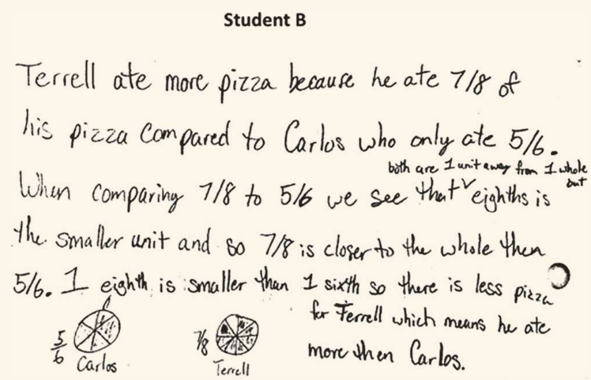
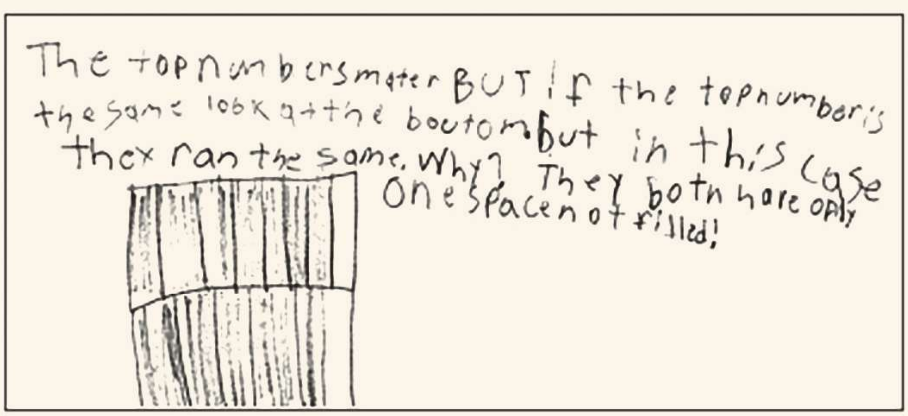



***Figure 2*** A Student Uses Part-whole Reasoning to Compare 5/6 and 7/8.



### Distinguishing Between Part-Whole and Fraction-As-Measure Conceptions

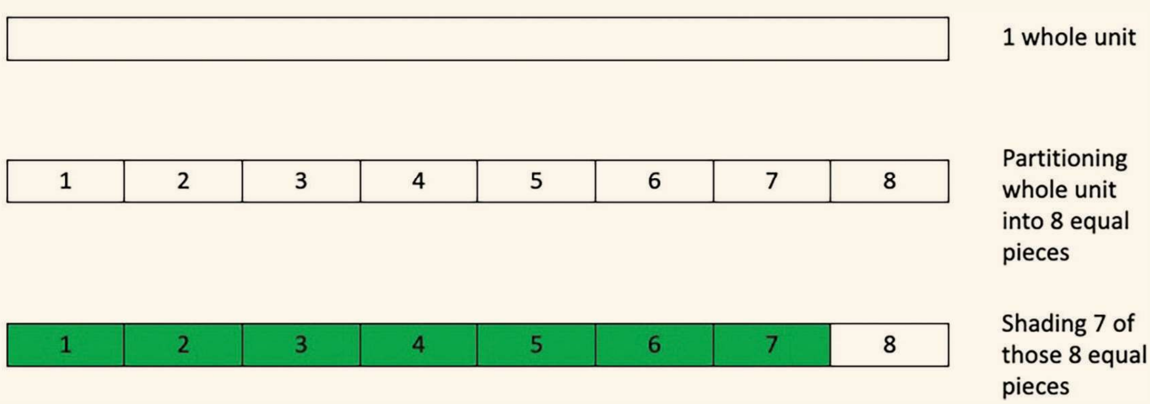
Kieren (1980) identified five subconstructs, or conceptions, related to fractions: part-whole, quotient, measurement, ratio, and operation. The focus of this paper is to distinguish between *part-whole* and *fraction-as-measure* conceptions. Some argue that a *part-whole* conception of fractions is the foundational fraction concept (Behr et al., 1983), with a *fraction-as-measure* conception an aspect of that conception. Lamon (2007), however, argued that instruction that builds *fraction-as-measure* conceptions best supports other conceptions of fractions, including part-whole conceptions. We are making an explicit distinction between these two conceptions. A student with a *part-whole* conception of fractions sees two quantities: the quantity of shaded parts and the quantity of total parts. Meanwhile, a student with a *fraction-as-measure* conception sees a quantity and a size of unit. It is not surprising that students with a part-whole conception would reason that the fractions 5/6 and 7/8 are equal because they are merely "seeing" the numerals "6" and "8" as quantities, not size of units.

### Why Might a Dominant Part-Whole Conception Develop?

We are defining a dominant conception as the concept image (Tall & Vinner, 1981) that a student most readily associates with a concept and/or utilizes when they are unsure how to reason. What is challenging is that many of the ways that fractions are modeled make a part-whole conception more visible than a fraction-as-measure conception. To illustrate this, consider the actions to create a linear model of the fraction 7/8 (see Figure 5). As the student did in Figure 2, the whole unit is partitioned into 8 equally sized parts, and 7 of those parts are shaded. The two quantities—7 shaded parts and 8 total parts—are visible. While the elements of a fraction-as-measure conception are present they are less visible. That is, a unit of *one* is partitioned into 8 equally-sized pieces, but each piece represents a unit length—1 *eighth*—in relationship to a unit of *one*. A length of 7/8 of a unit of *one* is 7 iterations—or copies—of the unit fraction, 1 *eighth*. The unit size is *eighths*—or 1 *eighth*—and it is spoken when we say the fraction name (e.g., "seven *eighths*"). However, students with a dominant part-whole conception of fractions only attend to the quantity of the whole to be shaded (7) and the total (8), not the size of the unit connected to the length of each shaded region (7 copies of a unit length of 1 *eighth* of a unit of length *one*).

Ensuring that part-whole conceptions of fractions do not become dominant in students' early work with fractions is challenging because many of the curricular resources teachers use in U.S. classrooms support the development of this conception (Simon et al., 2018). For example, an instructional strategy one of our colleagues formerly used in their classroom was to place discrete objects on a fraction tower. As shown in Figure 6, students' attention—consistent with the development of a part-whole conception of fractions—is drawn to the quantity of objects placed on the fraction tower and the total quantity of objects needed to complete the whole, not the size of the unit corresponding with the quantity of missing pieces. That is, one more object is needed

***Figure 5*** Using a Part-Whole Conception to Create 7/8



to make the whole for each fraction, not that the same quantity of missing pieces (i.e., 1) represents different sizes of lengths, or units (i.e., *sixths* and *eighths*, respectively).

These same curricula also often emphasize missing piece area models. These models illuminate elements of a part-whole conception. As shown in Figure 7, the elements of a part-whole conception are illuminated: 3 shaded parts and 8 total parts.

## WHY IS THIS PROBLEMATIC?

We believe students developing a dominant part-whole conception of fractions early in their experiences with fractions is problematic for three reasons: (1) it results in many students developing an invalid reasoning strategy, *gap reasoning,* that results in correct answers to a large number of common fraction comparisons; (2) it does not align with conception of fractions recommended by the the third- and fourth-grade fraction Common Core State Standards for Mathematics (CCSSM; National Governors Association Center for Best Practices & Council of Chief State School Officers [NGA Center & CCSSO], 2010), which emphasize the development of *fraction-as-measure* conceptions; and (3) it interferes with the development of a conceptual understanding of operations involving fractions (i.e., addition and subtraction).

### Invalid Reasoning Strategy That Works

Gap or "missing piece" reasoning leads to a correct answer *every time* students are asked to compare two fraction pairs of the third-grade CCSSM standards 3.NF.A3.D (NGA Center & CCSSO, 2010), that is, fraction pairs that have the same numerator or denominator. For example, consider comparing the same numerator unit fractions 1/3 and 1/4. A student using gap reasoning would correctly answer that 1/3 is greater than 1/4 because "1/3 is 2 away from the whole (1 + **2** = 3) while 1/4 is 3 away from the whole (1 + **3** = 4) so 1/3 is greater."

To further illustrate this point, consider a student using gap or "missing piece" reasoning to compare 7/8

***Figure 7*** Example of a "Missing Piece" Model

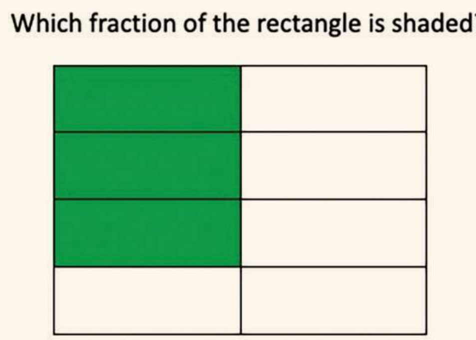
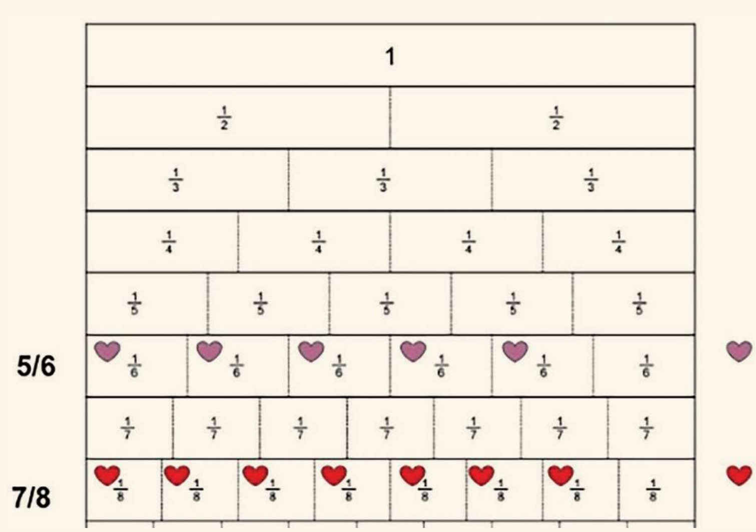



***Figure 6*** Using a Fraction Tower with Discrete Objects to Compare 5/6 and 7/8

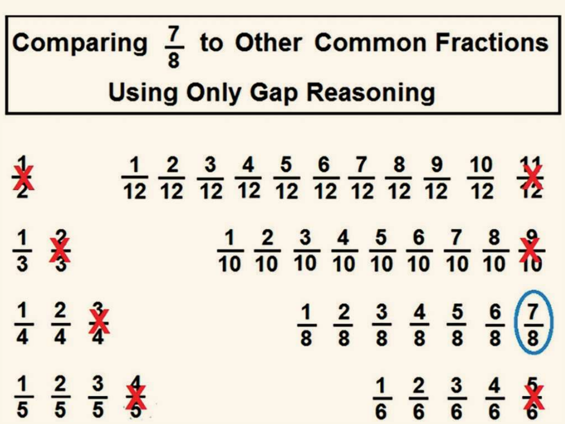


to other common fractions, as shown in Figure 8. The only fractions that students would incorrectly answer are those that are "1 away from the whole." In each of these instances, a student using gap or "missing piece" reasoning would indicate that the fraction pairs are "equal."

### Lack of Alignment With Common Core Standards

The results of our research suggest that many students begin using a part-whole fraction conception as early as third grade. A part-whole conception does not align with the fraction conception of the third-grade CCSSM (NGA Center & CCSSO, 2010), which states that students are to "develop an understanding of fractions as numbers" [3.NF]. Developing an understanding of fractions as a number aligns with a fraction-as-measure conception. Recall with part-whole reasoning that there is no size of unit, merely two quantities. To be seen as a number, students must see not only a quantity, but also a size of a unit. For example, consider a whole number: the placement of each numeral in the whole number "34" signifies both a quantity and a size of unit based on place value, 3 *tens* and 4 *ones*. Now consider those same numerals in the fraction 3/4. The "3" signifies the quantity and the "4" signifies the size of a unit, *fourths*. Up until fractions are introduced, students' conception of numerals has been in relation to quantities. The denominator of a fraction is the only instance in elementary mathematics in which a numeral represents a size of a unit.

***Figure 8*** Fraction Comparisons to 7/8 that Yield Correct Answers Using Gap Reasoning




### Interferes With Understanding Fraction Operations Conceptually

If your students have already had experiences with adding fractions, consider giving them the following problem: "Thomas ate 3/4 of a whole medium pizza, and Lydia ate 5/8 of a whole medium pizza. Together they ate how much of a whole medium pizza?" Given the familiarity of the context of the problem for most of our students, we were surprised by the significant number of students across all levels whom we had already identified as having dominant part-whole conceptions who answered 8/12. In other words, they added the quantity of eaten pieces (3 + 5 = 8) and the quantity of total pieces (4 + 8 = 12) instead of considering the need for the same size of unit to perform the action (i.e., 3 *fourths* + 5 *eighths* = 6 *eighths* + 5 *eighths* = 11 *eighths*). A fraction-as-measure conception, not a part-whole conception, underpins the process of fraction addition. Combining numbers (i.e., whole, fraction, and decimal) requires the size of the units to be the same. For example, the action of combining the two numbers 3/4 and 5/8 cannot be performed until the size of the units is the same. In this instance, they are not the same (i.e., *fourths* and *eighths*), so an *equal exchange* (Sullivan, 2023) of numbers (6 *eighths* for 3 *fourths*) is needed to complete the action. In other words, 3 *fourths* and 5 *eighths* cannot be combined, but 6 *eighths* and 5 *eighths* can be combined because they both have the same size of unit (see Figure 9).

While fractions greater than 1 are not a focus of this article, it seems important to note that a dominant part-whole conception of fractions does not support students reasoning about improper fractions either. For example, consider how a student utilizing a dominant part-whole conception of fractions would reason about the improper fraction 8/7. This would be challenge for them because it does not make sense, from a part-whole conception perspective, to have 8 shaded parts out of 7 total parts.

## INSTRUCTIONAL RECOMMENDATIONS

Our instructional recommendations are guided by our desire to develop dominant *fractions-as-measure* conceptions over *part-whole* conceptions. Recall that the third- and fourth-grade CCSSM (NGA Center & CCSSO, 2010) emphasize seeing fractions as numbers and placing fractions on a number line [3.NF & 4.NBT], both of which align with *fraction-as-measure* conceptions (Hackenberg, 2013). If students have already had experiences with fractions, it is important to first identify the nature of their current fraction conceptions using tasks similar to those shown in Figure 10.

The patterns of reasoning that students often show on these tasks are characterized in Figure 11. The notation (C) in the table indicates the student reasoning yielded a correct answer.

If students' responses exhibit reasoning that suggests either a part-whole conception or no-fraction

***Figure 9*** Model of the Fraction Addition Problem 3/4 + 5/8

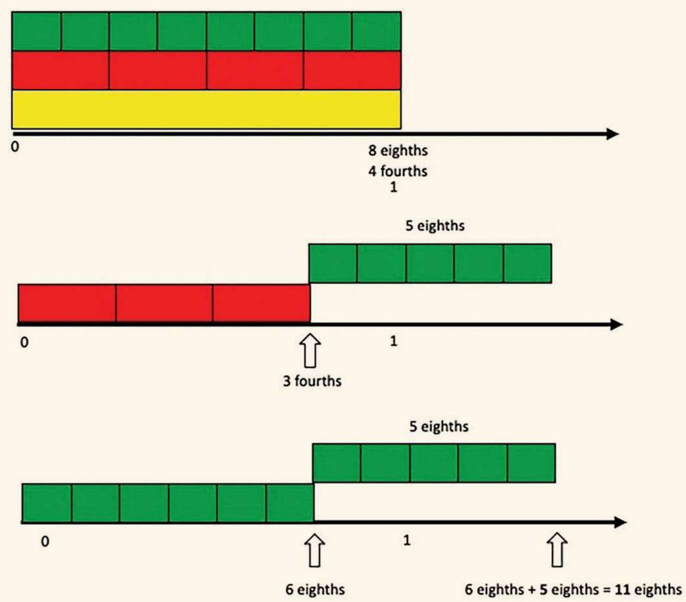
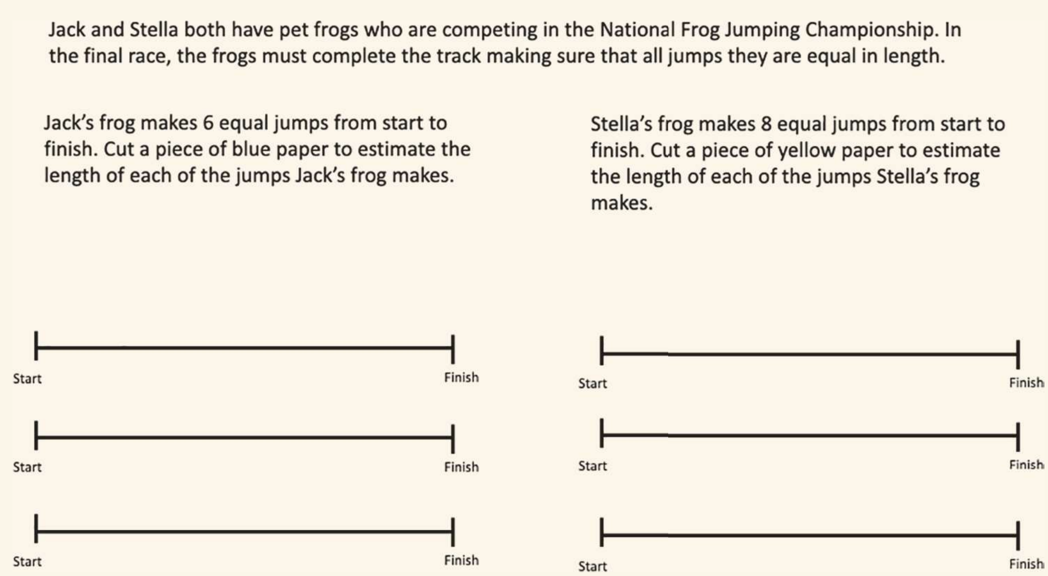



***Figure 10*** Tasks to Uncover Students' Fraction Conceptions



conception, there are two strategies that we use to explicitly focus students' attention on the size of the unit: (1) engaging students in partitioning and iterating activities that lead to cognitive conflict while also challenging students to engage in residual reasoning, and (2) using the numeral-unit-name notation to represent fractions, instead of standard fraction notation.

## Partitioning and Iterating Activity

Engaging students in partitioning and iterating activities supports the development of fraction-as-measure conceptions (Wilkins & Norton, 2018). These activities benefit all students but are especially critical for those who have already shown evidence of struggling with fraction concepts (Fuchs et al., 2013). A sample of student work associated with an iterating and partitioning activity related to Task 1 and 2 (Figure 11) is shown in Figure 12. This activity engaged students in partitioning by folding paper strips representing whole units into equally sized units (i.e., *sixths* and *eighths*). These strips were then folded so that only the unit fraction was visible. Then, students were asked to iterate the unit fraction the number of times required to reach a unit length of *one*. It is important to emphasize that the length of the unit fraction is formed in relation to a unit length of *one* (fraction-as-measure conception) instead of a number of pieces in relation to a total number of pieces (part-whole conception). For example, a unit length of 1 *eighth* is formed by partitioning a unit of *one* into into 8 equal lengths. Iterating 8 unit lengths of 1 *eighth* will result in the same length as a unit length of *one* (i.e., 8 x 1/8 = 1).

The cognitive conflict may not happen as students reason about Task 1. Students with a part-whole conception often state that 1/6 is greater than 1/8 because it was "5 away from the whole as compared to 7 away from the whole." Some students may also rely on visual inspection, "seeing" that a length of 1/6 appears greater than a length of 1/8. When this happened, we pressed them to reason about the quantity and size of unit; the language of a fraction-as-measure conception was exemplified by the student reasoning shown in Figure 13.

It is important to note that folding fraction strips into *eighths* (i.e., halves of halves of halves) is much more intuitive for students than *sixths* (i.e., thirds and halves). We have found it kept the focus on the main goal of the lesson if we showed students how to perform—at least initially—the folds to create sixths.

We have found that the cognitive conflict for students with dominant part-whole conceptions often

***Figure 11*** Different Conceptions Students Used to Solve the Jack and Stella Tasks

| Fraction Conception | Task 1 | Task 2 |
|---|---|---|
| No fraction conception<br><br>Attending to the magnitude of one of the whole numbers in the fraction. | "They are equal because both numerators are the same."<br>"Stella ran further because 8 is greater than 6." | "Stella ran further" because either "7 is greater than 5" or "8 is greater than 6." (C) |
| Part-whole conception<br><br>Gap reasoning | "Jack ran further because Jack is 5 away from the whole while Stella is 7 away from the whole." | "Stella and Jack ran the same amount because both are 1 away from the whole." |
| Partial Fraction-as-measure<br><br>Recognition of the different sizes of units | "Jack did because sixths are larger pieces than eighths." (C) | "Jack did because sixths are larger pieces than eighths." |
| Full Fraction-as-measure<br><br>Coordination of both the quantity and size of the unit<br><br>Residual reasoning | "Jack did because sixths are larger pieces than eighths and each ran 1 of those pieces" (C) | "Stella did because she is 1 eighth from the finish line and Jack is 1 sixth from the finish line. Eighths are smaller pieces than sixths, so she is closer to the finish line." (C) |

occurs with Task 2. Many of these students had already reasoned that Jack and Stella had run the same length. To move their thinking forward, we asked students to place the unit fractions 1/6 and 1/8 at the end of the unit length of *one,* as shown in Figure 14, and reason about who has a greater length to run to reach the finish line and how that information could be used to determine who had already run further.

### Using Numeral-Unit-Name Notation

As we have emphasized, the key to building fraction-as-measure conceptions is to focus students' attention on the meaning of the size of the unit. One way to do this when students are first introduced to fractions is to use *numeral-unit-name* notation (e.g., 7 *eighths*) to help students differentiate between a quantity and a size of unit. Brain research suggests that students intuitively conceptualize numerals as quantities (Sousa, 2016), and their previous mathematics experiences with whole numbers has reinforced this conception. The traditional fraction notation we use (e.g., 7/8) is the only time in which a numeral represents a size of unit (e.g., 7/**8** is 7 **eighths**). Using the unit name for the size of the unit maintains consistency with students' already-formed conception of numerals representing quantities (e.g., **8**0 is **8** tens and **8**/5 is **8** fifths and 5/**8** is 5 **eighths**). We also have found it disrupts students' reliance on part-whole conceptions of fractions because there is no second quantity with which to reason.

Writing out the unit name also provides an opportunity to focus students' attention on an important language acquisition concept that is connected to partitioning actions. Except for halves and thirds, the unit

***Figure 13*** Example of Student Reasoning About Task 1

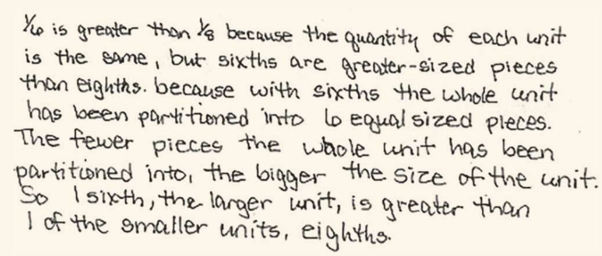
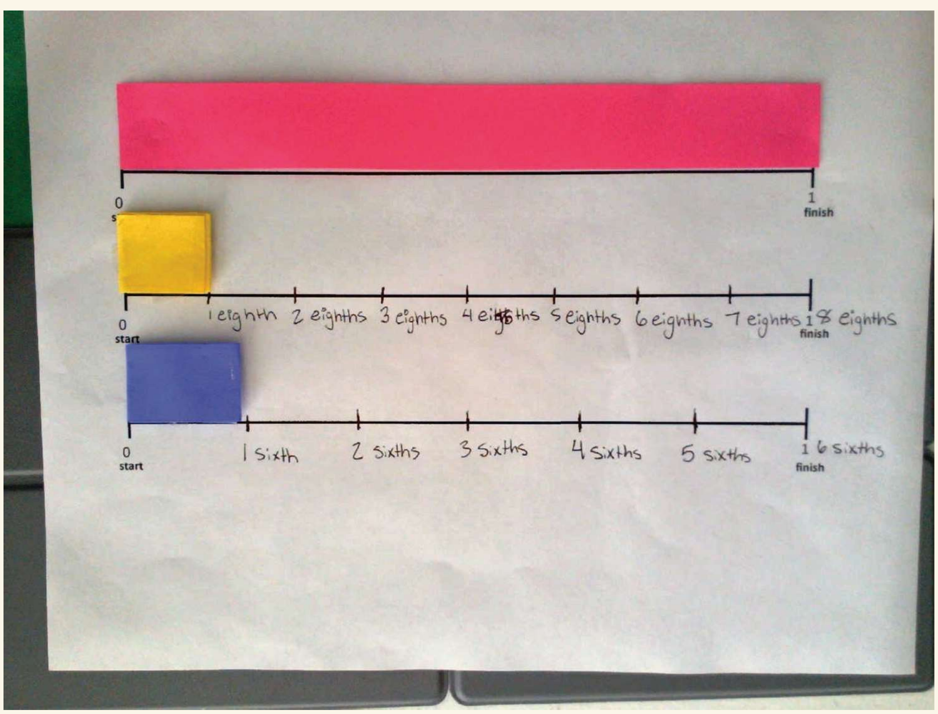



***Figure 12*** Example of Student Work Modeling Task 1



name for other size of units ends in the morpheme "*ths*" (e.g., four*ths*, six*ths*, eigh*ths*, twelf*ths*). In a fraction context, the morpheme "*ths*" means that the whole unit is partitioned into a quantity of equally sized lengths signified by the part of the word before "*ths*" (e.g., four, six, eight, twelve).

## CLOSING THOUGHTS

As mentioned earlier, our ability to emphasize fraction-as-measure conceptions is often hindered by curriculum that emphasizes part-whole conceptions over fraction-as-measure conceptions. As we did, we challenge you to review your own curriculum to make sure students are provided opportunities to engage in partitioning and iterating activities using paper strips and number lines.

Although our curriculum emphasized part-whole conceptions of fractions, we challenged students to use reasoning and language that supported fraction-as-measure conceptions. For example, when comparing fractions using area models like those shown in Figure 15, students would often reason that "2/6 is greater than 2/8 because I see that the pieces in 2/6 are bigger." Any time a student uses the word "piece," we challenged them to describe the size of the unit with respect to the unit of *one* making sure they communicated both the quantity and size of the unit.

Lastly—however tempting—we tried to avoid teaching students non-mathematical strategies (e.g., butterfly method) to compare fractions, we also delayed procedures (e.g., common denominator) until after we were certain that students had fraction-as-measure conceptions. It has been our experience, consistent with the findings of Pesek and Kirshner (2000), that once students have a strategy that they believe yields correct answers, they will rely on that strategy, often dismissing our attempts to build more conceptual-based strategies.

***Figure 14*** Illustrating How Much is Left to Reach the Whole

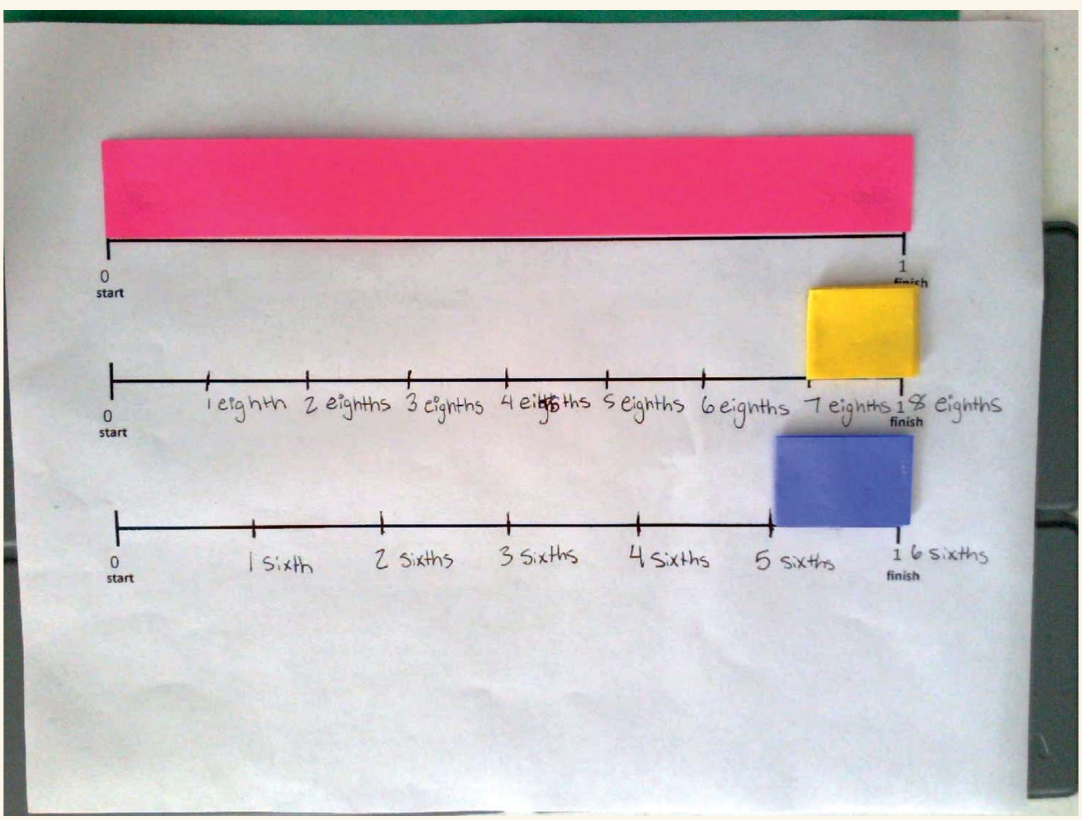
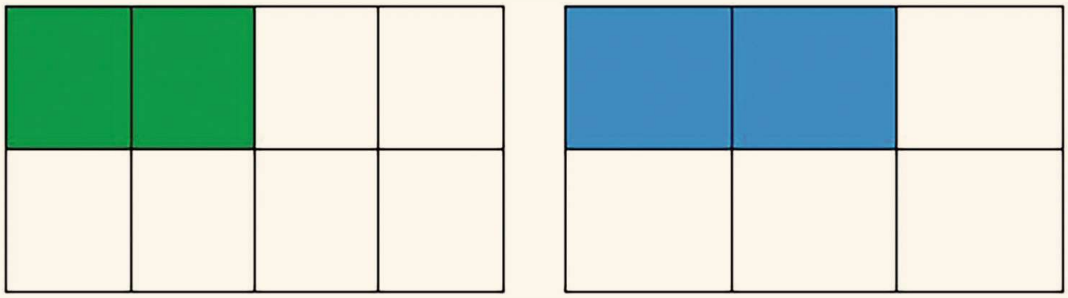


***Figure 15*** Example Using "Missing Piece" Models to Compare 2/6 and 2/8

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